**Affine Invariant Parameterization to Assess Local Shape in Abdominal Organs**

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**ABSTRACT**

We present a novel method for three-dimensional (3D) shape parameterization. The approach is affine invariant and is applied to comparing local shape across abdominal organs. The inherent structure of the abdominal organs is used to generate a regular sampling of the organ’s surface. ‘Planar-convexity’ is defined for a general 3D closed object as the property that there exists a set of parallel planes which cover the 3D space such that every intersection of a plane with the object is a singular closed planar curve. We show that this parameterization, combined with the 3D analogue of a 2D shape descriptor, successfully shown to be invariant under affine transformations and noise, can be effectively used to compare features of two closed 3D surfaces point-to-point. The technique avoids common problems with the parameterization of concave surfaces and shows great potential for analyzing and improving the automatic modeling and segmentation of abdominal organs.

**Keywords:** Abdominal organs, shape analysis, 3D parameterization, affine invariance.

1. **INTRODUCTION**

The volume and shape of abdominal organs from radiographic data are used to assess the presence of disorders, to make surgical decisions involving organ transplantation and to estimate the progress of disease-related treatments [1]. While the automatic segmentation of organs from medical scans can provide this information, computer-aided techniques generally struggle in the presence of abnormal anatomical and/or physiological features. It has been noted that it is beneficial to compare the shapes of analyzed patient organs to those of a training set in guiding segmentation techniques [2,3].

To identify subtle shape differences between two comparable objects, a robust parameterization of objects is required. As a simple example, the accuracy of an automatic segmentation is generally determined by comparing its performance against that of manual segmentations or ground truth. More interestingly, shape descriptors from a training database can be used to constrain the automatic segmentation of human organs from medical images. Previous methods for 3D surface analysis have been proposed in medical applications [4]. In particular, the method introduced in [5] involves generating a bijective mapping from the 3D surface to the unit sphere. Alternatively, spherical harmonics-based parameterizations divide a given surface into a set of basis functions; spherical harmonics have been applied to medical data with promising results [6]. Nevertheless, a reliable parameterization of the surfaces of human organs has yet to be made.

We present a novel technique to parameterize 3D surfaces, like those of abdominal organs, for the comparison of objects by a point-to-point correspondence. 3D objects are represented by parallel planes that intersect the surface of objects in closed planar curves. This spatial representation avoids common problems with the surface parameterization.
of concave objects. To compare local shape features of two organs we employ a shape descriptor invariant under rotation, scale, and noise. The method can successfully differentiate local and global shape differences between two organs and has the potential to improve organ segmentation by identifying erroneously segmented areas on the organ’s surface.

2. METHOD

2.1 Data

Eight CT abdominal scans were collected on LightSpeed Ultra and QX/I scanners [GE Healthcare] from patients without imaging artifacts in the abdominal organs. Image resolution ranged from 0.62 to 0.82 mm in the axial view with a slice thickness of 5 mm. Manual and automatic segmentations were obtained for the eight sets of livers, spleens, and left and right kidneys. The automatic segmentations were generated using a graph cuts-based algorithm [7]. Additional manual segmentations were available for stomachs and pancreases.

2.2 3D Shape Features

We chose to use a 3D-analogue of a curvature-feature as shape descriptor [8] for comparing closed planar curves, which we refer to here simply as the shape feature (Figure 1). This shape feature at a given point $p$ on a planar curve is the area of intersection of the interior of the curve and a ‘seed’, which is itself a closed planar curve of high regularity centered at $p$ (a circle was used previously in 2D). This shape feature $S$ (Equation 1) of a curve $C$ can be used for both local (at every point on two curves $f$ and $g$, as illustrated in Figure 1) and global comparison of two test curves, given an adequate simultaneous parameterization of the two curves. To extend the shape feature to 3D, a sphere was used as the seed.

$$S = \int_C V_r(p,x)dx;$$

$V$ is the volume of the intersection of the tested object and the seed sphere. The radius $r$ of the seed is adapted to the size of the analyzed object and $S$ is normalized by the volume of the seed, as shown in Equation 2. A compensation function was employed to account for the pixilation of digital objects. The shape descriptor is rotation, scale and noise invariant [8].

$$S = \frac{\int_C V_r(p,x)dx}{\int_{3D} V_r(p,x)dx};$$

2.3 3D Surface Parameterization

To allow point-to-point comparisons across multiple surfaces, our method uses the structure of a general class of objects we call ‘planar-convex’. We assume that abdominal organs fall under this category. We define a planar-convex object $O$ in $\mathbb{R}^3$ to be a closed surface for which a set of parallel hyper-planes $P$ exists, such that every intersection of a plane in $P$ with $O$ results in a singular closed planar curve. We call each set of parallel hyper-planes which intersects with $O$ in this way ‘convexity planes’. Some 2D examples are shown in Figure 2.

From principle component analysis [9] we first orient about the object’s largest component (main axis) a set of symmetric points comprising the vertices of a pentakis dodecahedron [10] (Figure 3). These dodecahedron points are rotated by the angle between the main axis and the z-axis $(0,0,1)$. We do this for both objects and align their main axes, and then likewise their dodecahedron points, as in Figure 4. This allows us to find multiple sets of parallel planes that intersect the object with normals ranging uniformly over a hemisphere. For this work we sampled over a 32-vertex dodecahedron.
Figure 1: An illustration of the shape descriptor on 2D closed curves. The seed is a circle that intersects the objects (here a square and a larger circle). Given a correct parameterization of the two closed curves, their point-to-point local difference can be assessed.

Figure 2: Objects represented in the planar-convex space. Objects include the simpler ‘V’ shape (top), a non-star shape domain such as the ‘S’ shape (middle), and the more irregular ‘ink blot’ shape (bottom). Convexity planes are colored blue, while their intersections with the respective objects are shown in red.
Figure 3: Surface sampling using spherical models; rays are shown from the centroid of the object at even angles on the left image, resulting in oversampling at poles; a uniform sampling is achieved using the vertices of a pentakis dodecahedron on the right image.

Figure 4: Alignment of points on dodecahedrons. A dodecahedron point $s$ is centered about the point $p$ of the main axis by rotating it by the angle $X$ between $p$ and $(0,0,1)$. Next, using the ratio of the largest x/y/z to smallest x/y/z pixel in a plane perpendicular to the object’s main axis (which we call mother plane), we determine which primary plane (x, y or z-plane) maps injectively to the mother plane. We then project this primary plane onto the mother plane (Figure 5). Each mother plane is translated along its normal, making a bounding box for the object, and $n$ equidistant parallel planes determine the intersections of the translated mother plane and the object. Each intersection is then analyzed for the number of 2D connected components it contains. The process is repeated for the other axes of the objects and their corresponding set of parallel planes, from which the average number of connected components is found.

Figure 5: The mother plane (blue) and its projection from the primary plane (red and green).
The minimum sum of average components across the corresponding axes/planes of the two compared objects is then computed. This defines the set of convexity planes $P$ (as defined above) selected for both objects. The surface of each convexity plane is uniformly sampled by a user-defined number of partitions, with points at these partitions being projected onto the object surface (Figure 6). These projections or ‘parameterization points’ have point-to-point correspondence between the compared objects/organs.

At each parameterization point we then compute the shape feature $S$. Features are compared locally at the corresponding pairs of intersection points on the two objects, while the histograms of shape features over the entire objects allow a global analysis. Finally, shape features are mapped onto both objects’ surfaces to allow the visualization of results.

3. RESULTS

The 3D parameterization is robust on a variety of shapes from abdominal organs. The convexity planes are retrieved on organs with intricate concave shapes, such as liver, pancreas or stomach (Figure 7). The planar-convexity property is employed for the parameterization of abdominal organs and finding point-to-point correspondences between two shapes. This property results in an affine invariant 3D shape feature that can identify local and global shape differences between two organs. An example of 3D parameterization of a liver surface is presented in Figure 8.

For the intuitive presentation of results, shape difference maps are shown projected onto each of the compared objects. A ‘hot-cold’ colormap presents small local shape differences in cold colors and larger discrepancies in hot nuances. The efficacy of the shape feature and its parameterization is illustrated in comparisons between manual and automated segmentations of liver (Figure 9). The method can identify the most similar organs from a training database to the patient data and has the potential to guide statistical and shape-based segmentation and modeling methods for computer-aided diagnosis.

4. CONCLUSION

We presented a novel method for comparing local shape across abdominal organs. The liver, spleen, kidneys, pancreas, and stomach were shown to be planar-convex; for each organ there exists a set of parallel planes such that every intersection of a plane and the organ is a closer planar curve. Furthermore, we have shown how this characteristic can be used to generate an effective parameterization of the surface of abdominal organs. Along with the shape feature, we showed evidence that this parameterization technique effectively allows point-to-point comparison of local shapes.

The robust parameterization combined with an invariant shape feature will allow identifying organs (or parts of organs) from a training database that are similar to a given patient’s data. Similarity scores are essential in statistical modeling.
and segmentation algorithms for abdominal data. Using a patch generator framework similar to the one developed in [11] will allow comparing regions of an organ/object to corresponding training patches defined by anatomical or morphological criteria.

**Figure 7:** Convexity planes are presented for: top row – right kidney (left), spleen (middle), and liver (right); and bottom row -- stomach (left) and pancreas (right). All organs are represented as planar-convex objects in $\mathbb{R}^3$.

**Figure 8:** Parameterization points are highlighted as small cubes on the surface of a liver with irregular shape. These points allow point-to-point correspondence between two shapes.
Figure 9: A manual (left) segmentation of the liver is compared to the automatic (right) segmentation of the same organ; their volume renderings are shown on the top row. The top 50% of differences are mapped in red to both objects’ surfaces on the bottom row. Main differences are identified around the inferior vena cava and on the left lobe of the liver.

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REFERENCES


